

Interaction between large scales and **temperature** fluctuations during the 2003 heat wave

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Outline

1. The question

Context of blocking

Problem: – each blocking is unique

– unravel the mechanisms of genesis, persistence and dissipation of each blocking

2. Methodology:

First principles + Triple decomposition framework

– time derivatives of temperature variance

– highlight the **interactions** between large (quasi-periodic) scales and **small-scale statistics** during the 2003 heat wave

3. Results

Data analysis (2nd- and 3rd-order moments), summer 2003, ERA5

1. Context – I. The general context of climate

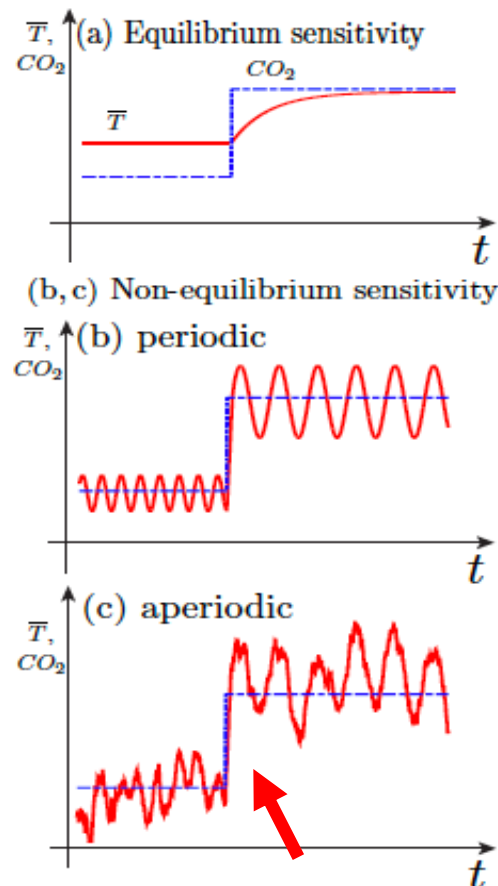
Climate and Its Sensitivity

Let's say CO_2 doubles:

How will “climate” change?

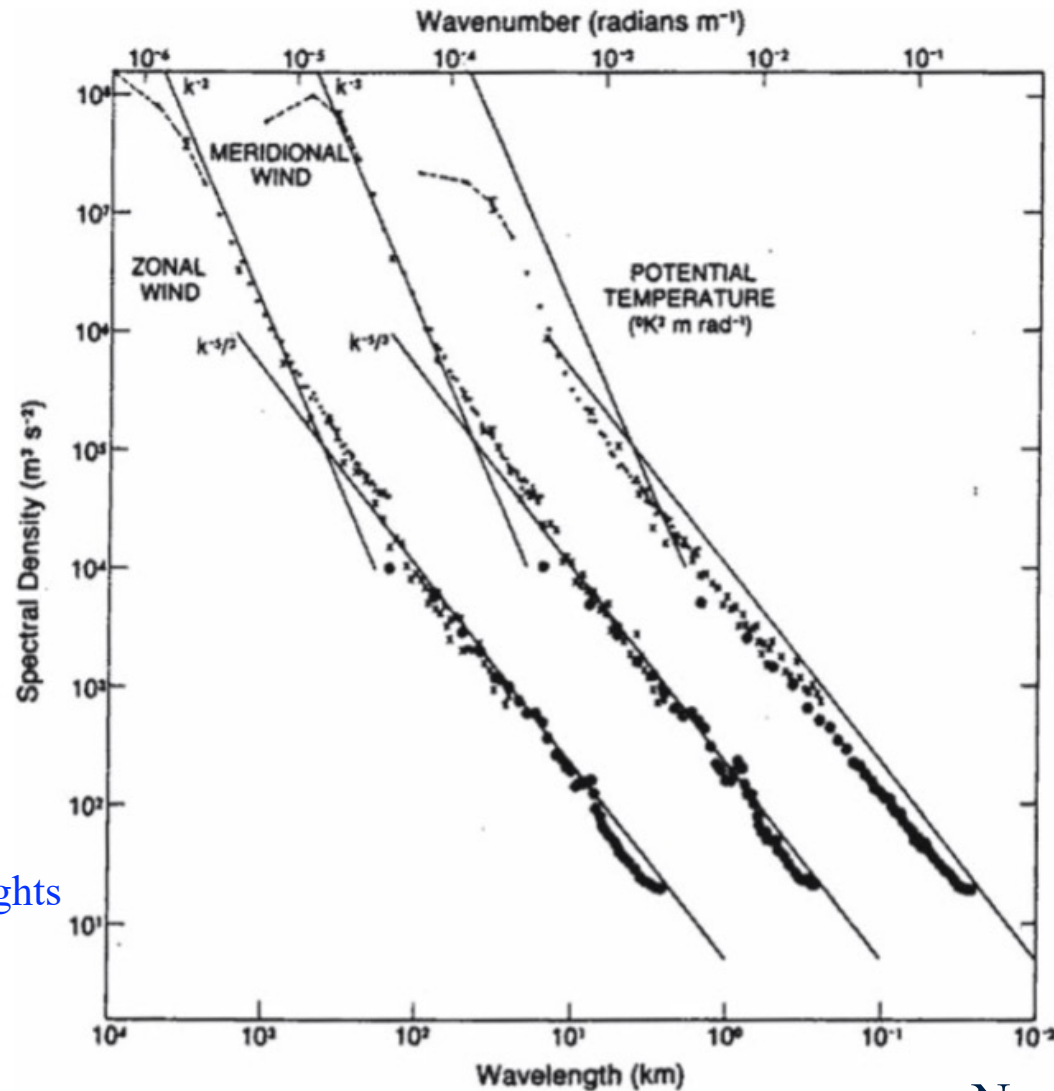
1. Climate is in **stable equilibrium** (fixed point); if so, **mean temperature** will just shift gradually to its new equilibrium value.
2. Climate is **purely periodic**; if so, **mean temperature** will (maybe) shift gradually to its new equilibrium value. But how will the **period, amplitude and phase** of the **limit cycle** change?
3. And how about some “real stuff” now: **chaotic + random**?

Ghil (in *Encycl. Global Environmental Change*, 2002)



Jump of fluctuations and statistics:
Need for $d/dt (M_n)$
Here: $n = 2$, focus on production

1. Context – II. Scales: MacroTurbulence

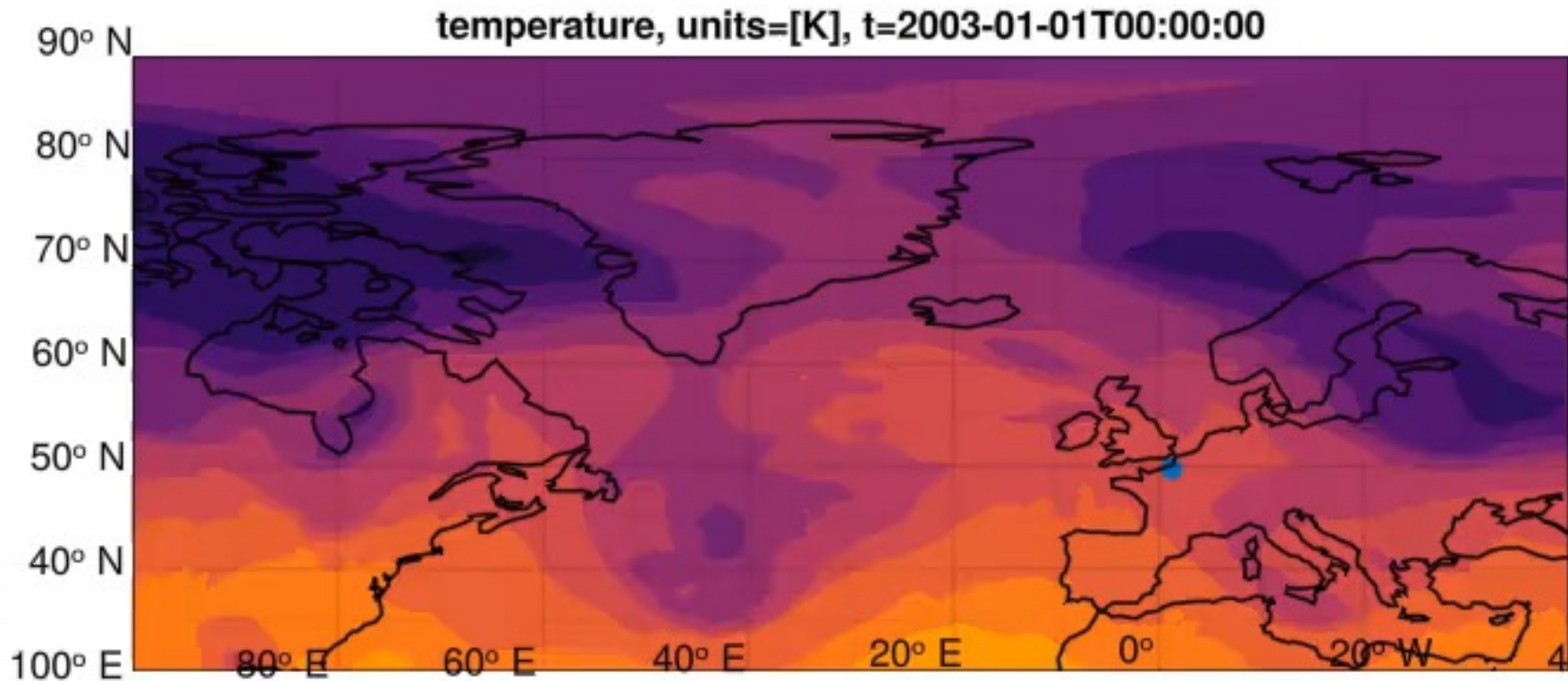


Nastrom & Gage (1985)

- Obs. from commercial flights
- 9–12km altitude

All scales are present: different scalings, reflecting different physical mechanisms

1. Context – III. Blocking vs. heat waves, summer 2003



Hourly 500 hPa Temperature time series for 1 Jan. – 31 Dec. 2003

5

All scales are present: The MacroTurbulence

2. Methodology – I. Transport equations

The approach. Phase averages

Triple decomposition³: $\beta = \bar{\beta} + \tilde{\beta} + \beta'$

Phase-average: $\langle \beta \rangle = \bar{\beta} + \tilde{\beta}$

Phase-averaged Strain: $\langle S \rangle = \bar{S} + \tilde{S} = \frac{1}{2} \left(\frac{\partial \langle U \rangle}{\partial y} + \frac{\partial \langle V \rangle}{\partial x} \right)$

³ Reynolds and Hussain 1972

Thiesset and Danaila, J. Fluid Mech. 2013,2014, 2020

Bouha, PhD thesis, 2016

Barbano et al., Bdry. Layer Met., 2022

Finnigan and Einaudi...

2. Methodology – II. Transport equations

2.1. Transport equations for T , $\bar{\theta}$ and $\overline{\theta'\theta'}$

The starting point is the heat transport equation

$$\frac{\partial \Theta}{\partial t} + U_j \frac{\partial \Theta}{\partial x_j} = \kappa \frac{\partial}{\partial x_j} \frac{\partial \Theta}{\partial x_j} \quad (2.1)$$

Following the triple decomposition of Reynolds & Hussain (1972), the velocity and temperature can be written as

$$\Theta = T + \bar{\theta} + \theta' \quad (2.2a)$$

$$U_j = \bar{U}_j + \bar{u}_j + u'_j \quad (2.2b)$$

Substituting (2.2) into (2.1), and then phase averaging, we obtain

$$\frac{\partial \bar{\theta}}{\partial t} + \bar{U}_j \frac{\partial T}{\partial x_j} + \bar{U}_j \frac{\partial \bar{\theta}}{\partial x_j} + \bar{u}_j \frac{\partial T}{\partial x_j} + \bar{u}_j \frac{\partial \bar{\theta}}{\partial x_j} + \left\langle u'_j \frac{\partial \theta'}{\partial x_j} \right\rangle = \kappa \frac{\partial}{\partial x_j} \frac{\partial (T + \bar{\theta})}{\partial x_j} \quad (2.3)$$

The time average of (2.3) gives the equation for the mean temperature field:

$$\bar{U}_j \frac{\partial T}{\partial x_j} + \overline{\bar{u}_j \frac{\partial \bar{\theta}}{\partial x_j}} + \overline{u'_j \frac{\partial \theta'}{\partial x_j}} = \kappa \frac{\partial}{\partial x_j} \frac{\partial T}{\partial x_j} \quad (2.4)$$

2. Methodology – III. Transport equations

The transport equation for $\theta'\theta'$ is

$$\frac{\partial \frac{1}{2} \overline{\theta'\theta'}}{\partial t} + \overline{U}_j \frac{\partial \frac{1}{2} \overline{\theta'\theta'}}{\partial x_j} + \frac{\partial \frac{1}{2} \overline{\tilde{u}_j \langle \theta'\theta' \rangle}}{\partial x_j} + \overline{\theta' u'_j} \frac{\partial T}{\partial x_j} + \overline{\langle \theta' u'_j \rangle} \frac{\partial \bar{\theta}}{\partial x_j} + \frac{\partial \frac{1}{2} \overline{u'_j \theta' \theta'}}{\partial x_j} = \kappa \frac{\partial}{\partial x_j} \frac{\partial \frac{1}{2} \overline{\theta'\theta'}}{\partial x_j} - \kappa \frac{\partial \overline{\theta'}}{\partial x_j} \frac{\partial \overline{\theta'}}{\partial x_j}$$

$$\frac{\partial \frac{1}{2} \overline{\tilde{\theta}\tilde{\theta}}}{\partial t} + \overline{U}_j \frac{\partial \frac{1}{2} \overline{\tilde{\theta}\tilde{\theta}}}{\partial x_j} + \overline{\tilde{\theta} \tilde{u}_j} \frac{\partial T}{\partial x_j} + \frac{\partial \frac{1}{2} \overline{\tilde{\theta}\tilde{\theta} \tilde{u}_j}}{\partial x_j} + \frac{\partial \overline{\tilde{\theta} \langle \theta' u'_j \rangle}}{\partial x_j} - \overline{\langle \theta' u'_j \rangle} \frac{\partial \tilde{\theta}}{\partial x_j} = \kappa \frac{\partial}{\partial x_j} \frac{\partial \frac{1}{2} \overline{\tilde{\theta}\tilde{\theta}}}{\partial x_j} - \kappa \frac{\partial \tilde{\theta}}{\partial x_j} \frac{\partial \tilde{\theta}}{\partial x_j}$$

3. Results – I: August 2003

Here, we show:

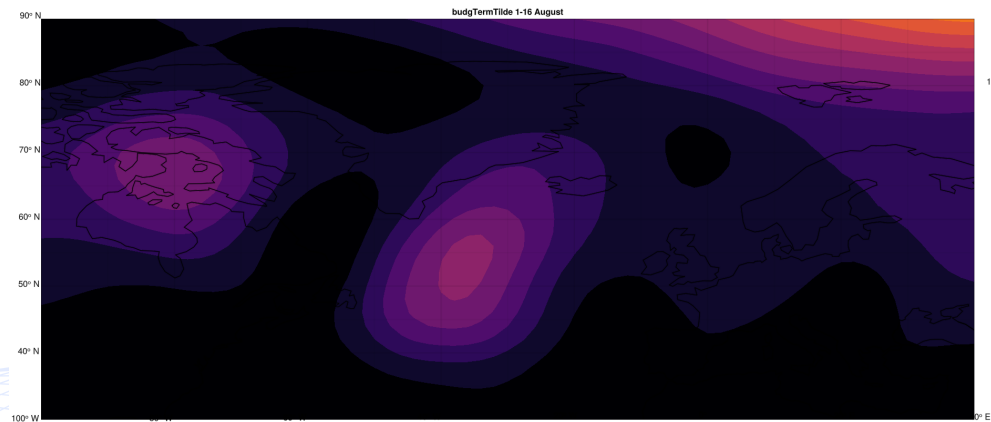
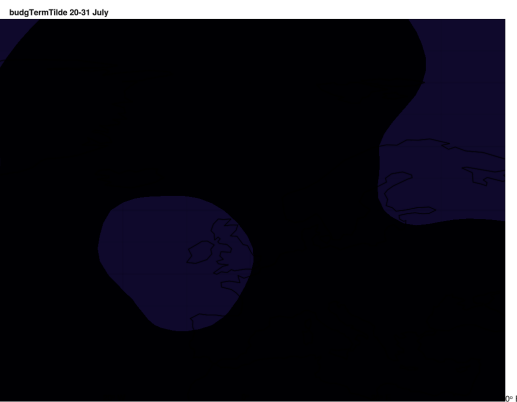
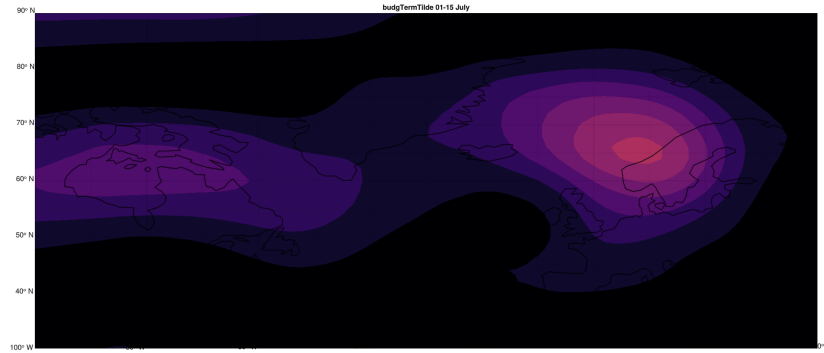
Decomposition **EMD** space-time (Huang)

One-point statistics

Variance (CM, RM)

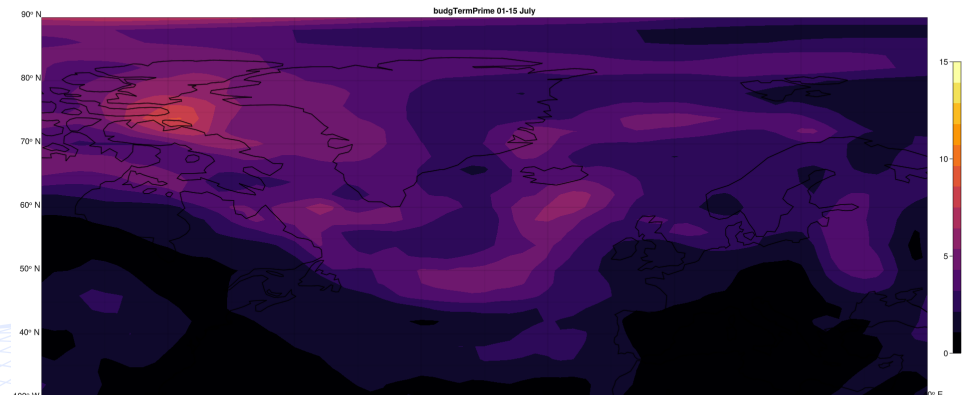
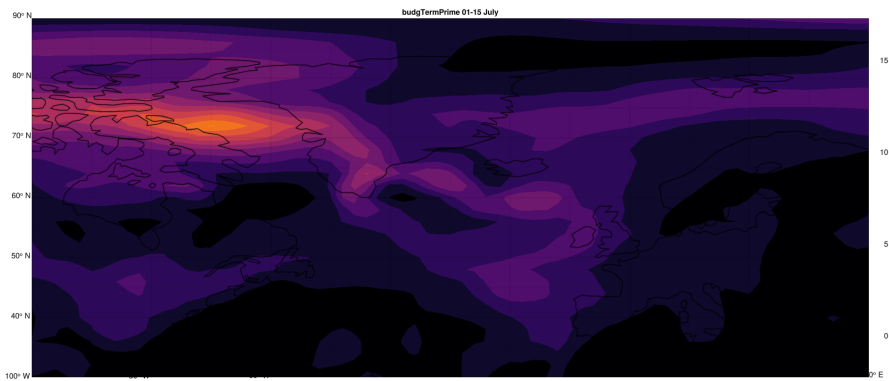
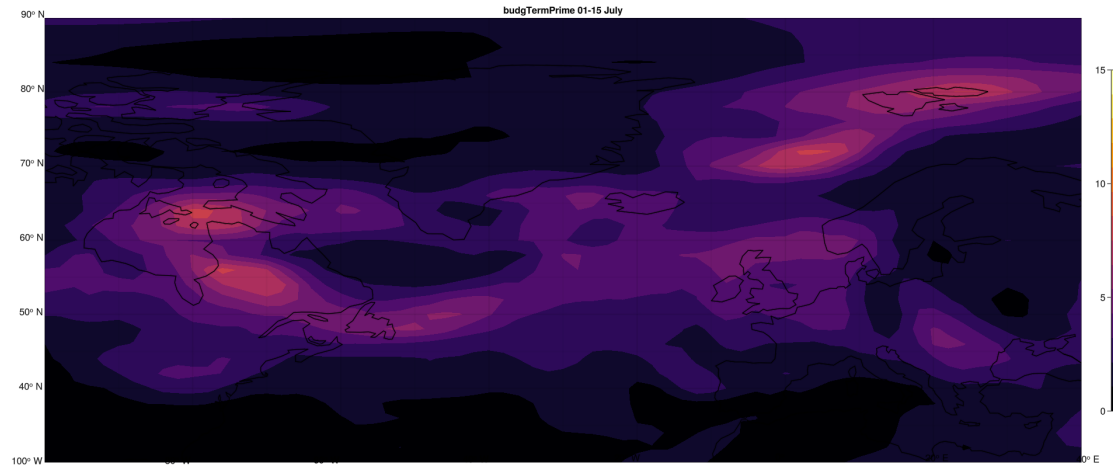
Production terms (CM and RM), Advection,

3. Results: Variance of CM temperature, periods 1, 2 and 3



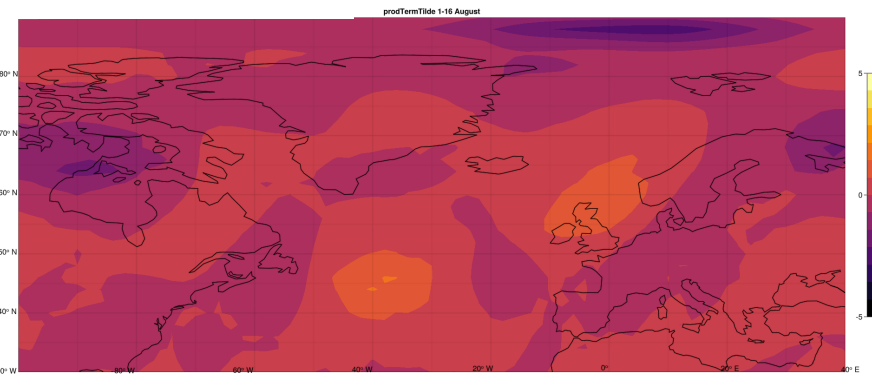
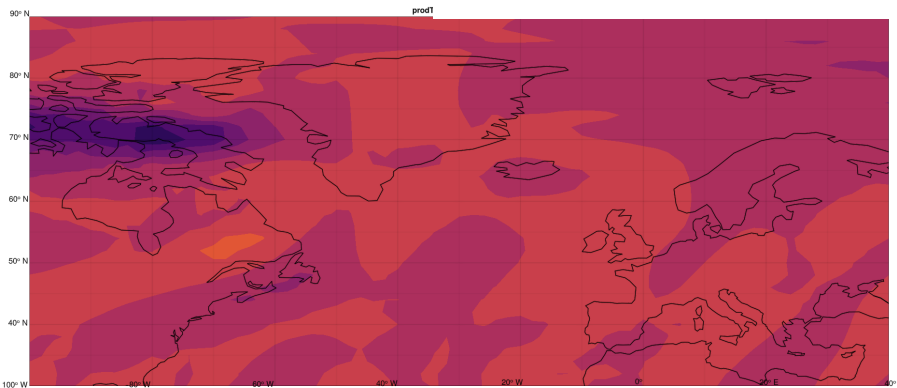
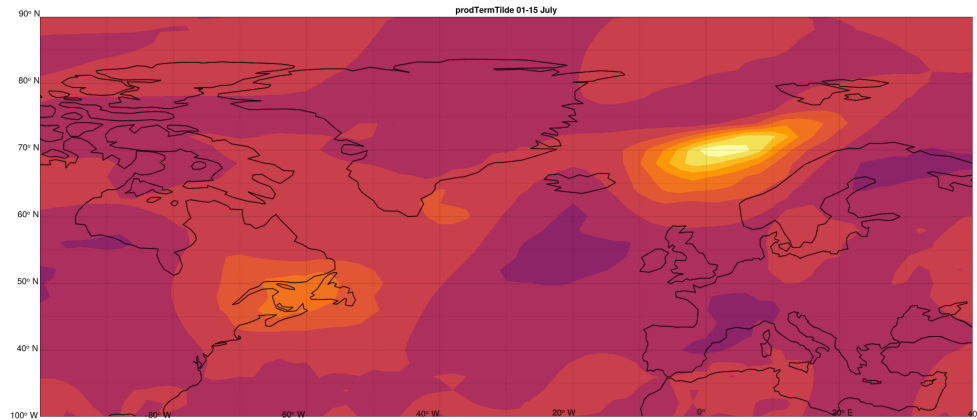
Less and less fluctuations for Europe (both CM and RM)

3. Results: Variance of RM temperature, periods 1, 2 and 3



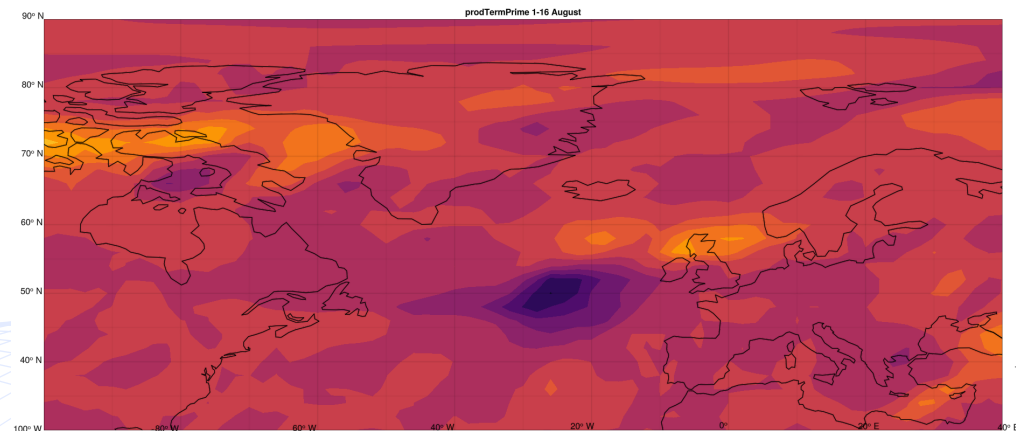
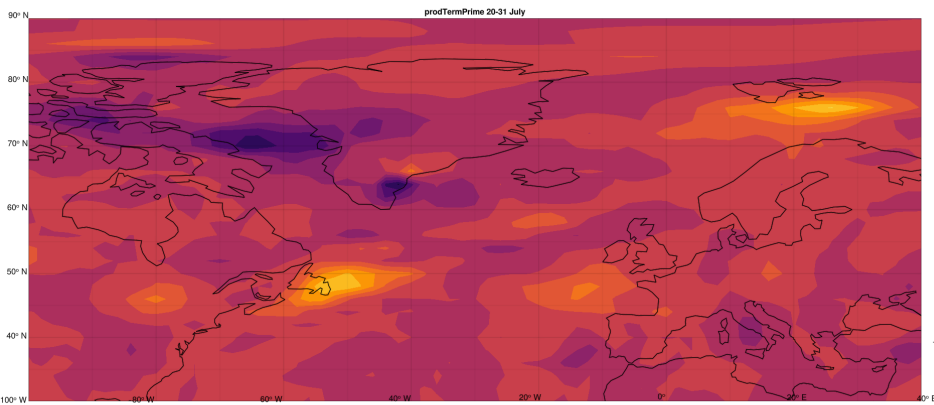
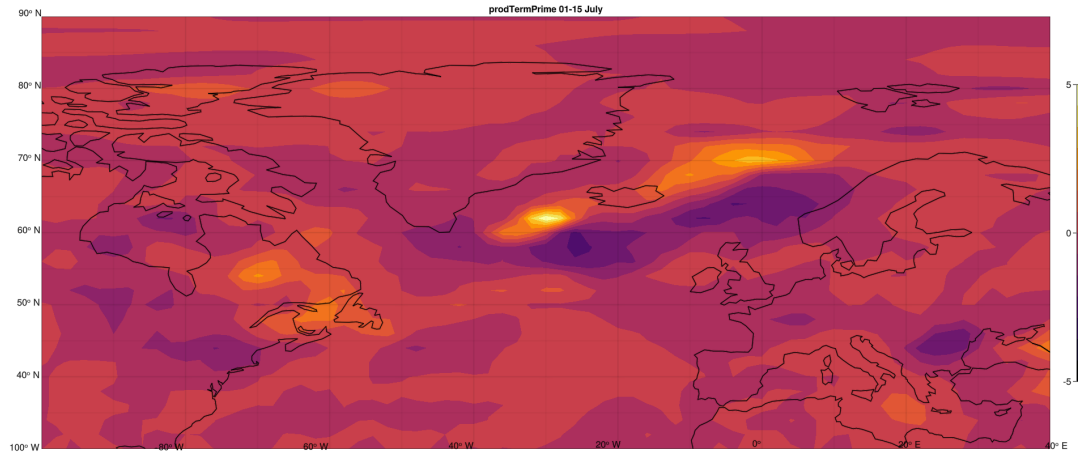
Less and less fluctuations for Europe (both CM and RM)

3. Results: Production of Coherent for 3 periods



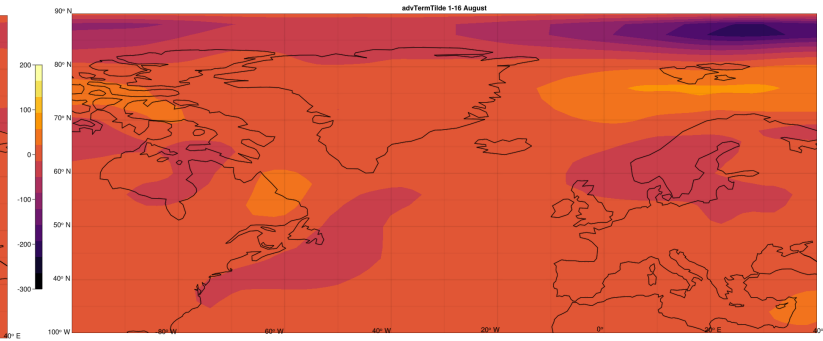
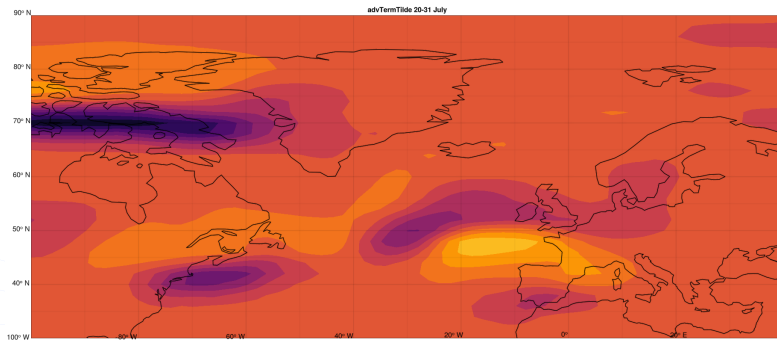
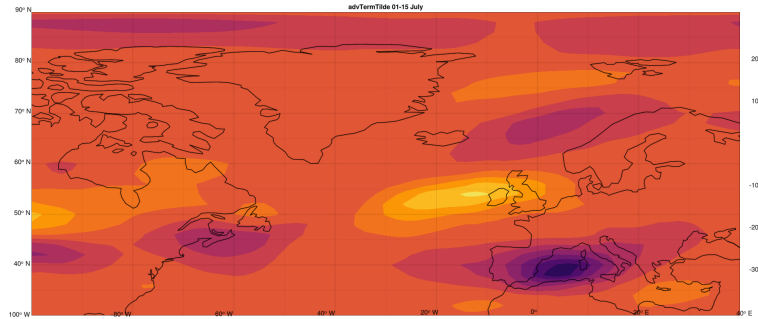
Production which is both positive and negative (sink-like)
Smaller and smaller (absolute) values of the production towards August

3. Results: Production of Random for 3 periods



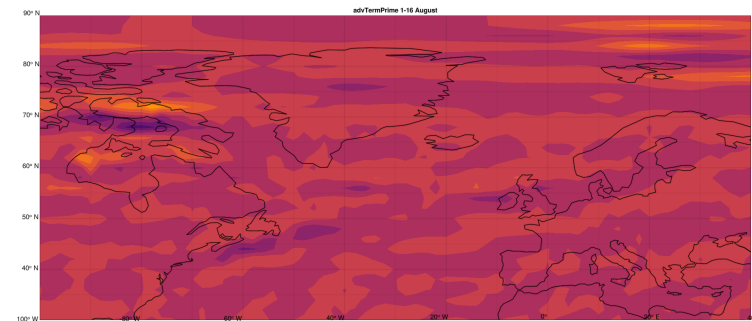
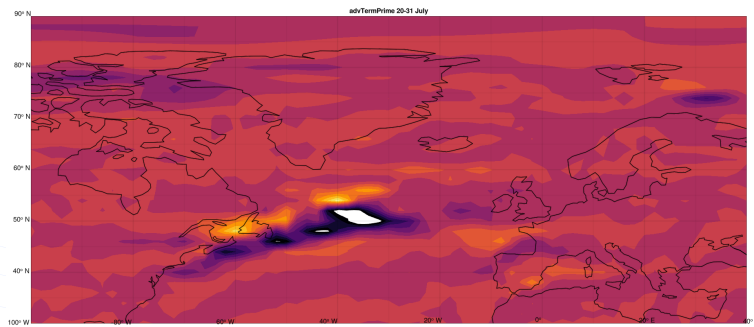
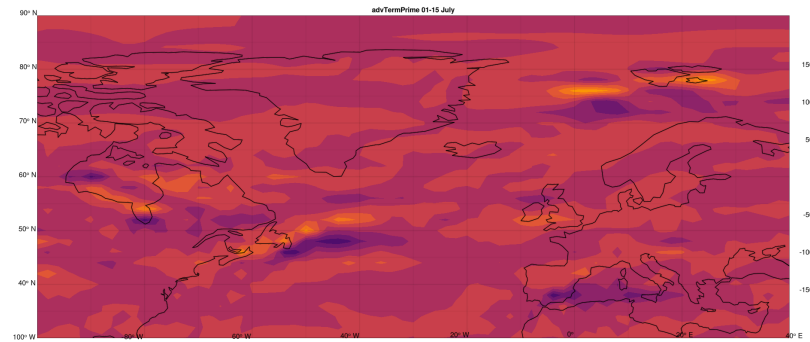
Larger values and more rapid dynamics for RM
Production which is both positive and negative (sink-like)

3. Results: Advection of Coherent motion



Smaller and smaller values towards August
Spatial extent of the motions CM

3. Results: Advection of Random motion



Smaller and smaller values towards August
Spatial extent of the motion, reduced over small scales

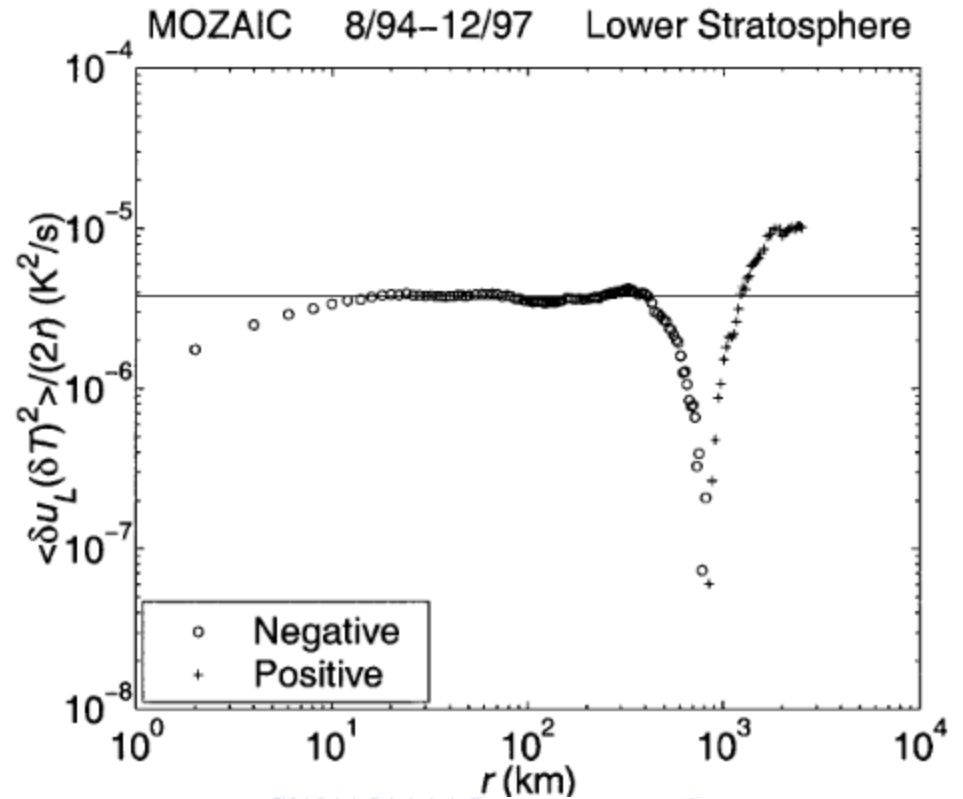
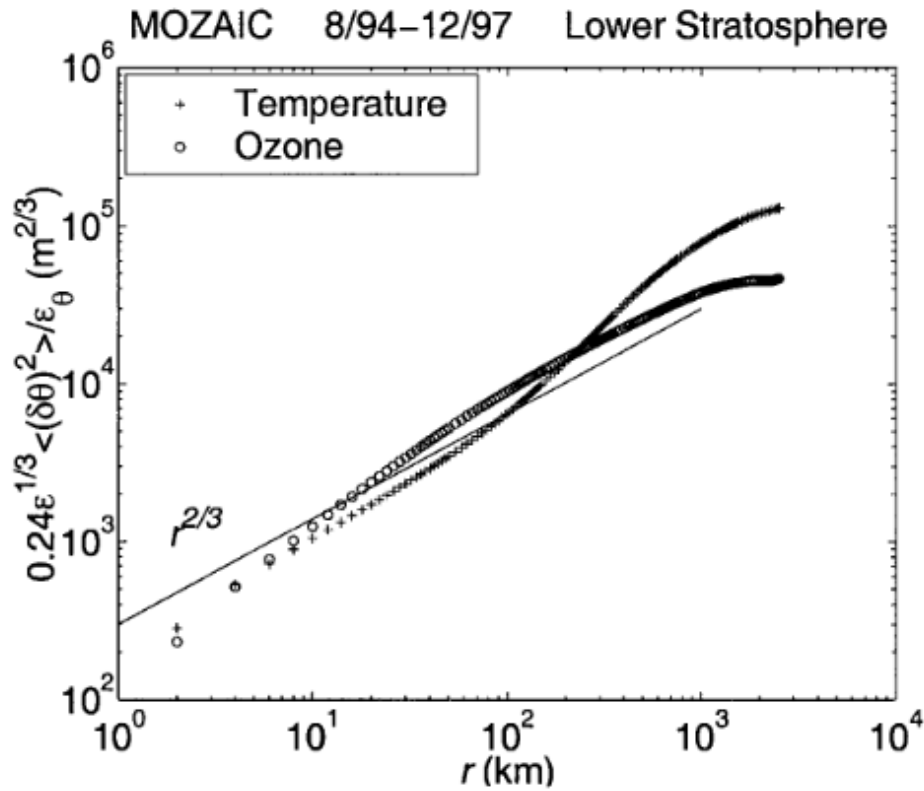
4. Conclusions

Key results

- Decomposition of the motion in CM and RM (EMD)
- Transport equations for CM and RM of temperature at 500 hPa
- Production term, both positive and negative (sink)

- Both 2nd- and 3rd-order structure functions disagree with Nastrom & Lindborg observations (difference in altitude and the use of Taylor hypothesis).

3. Results

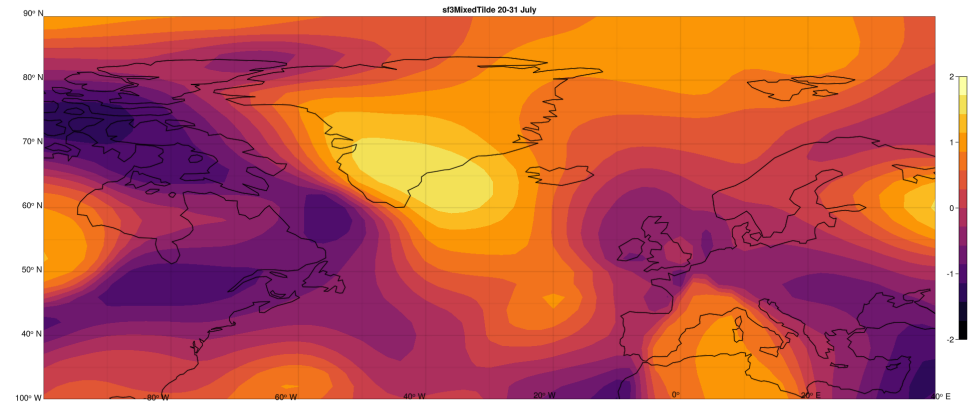
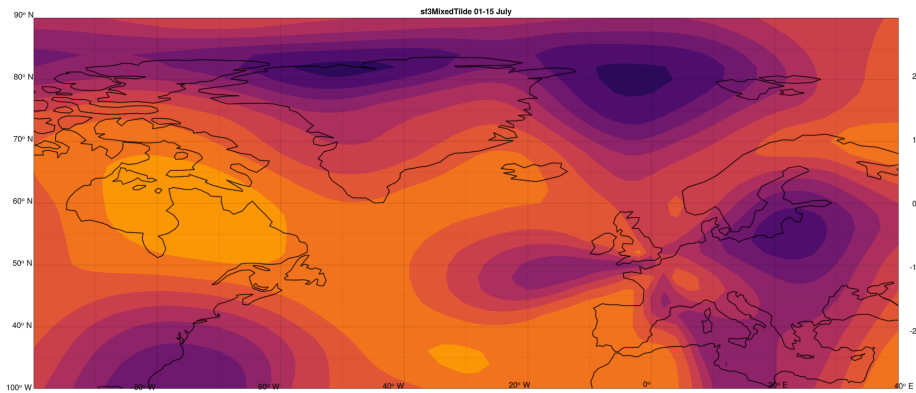


- Obs from commercial flights
- 9-12km altitude
- Temperature derived from velocity via Taylor hypothesis

(Linborg and Cho, 2000) 18

3. Results

Third-order mixed Structure functions for July 2003 (left) and August 2003 (right)

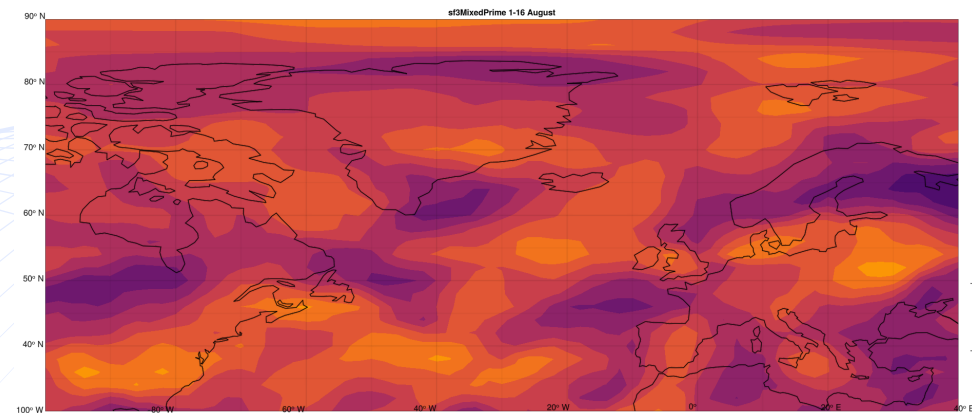
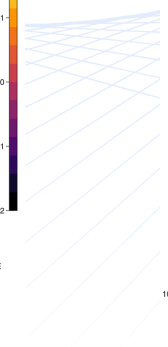
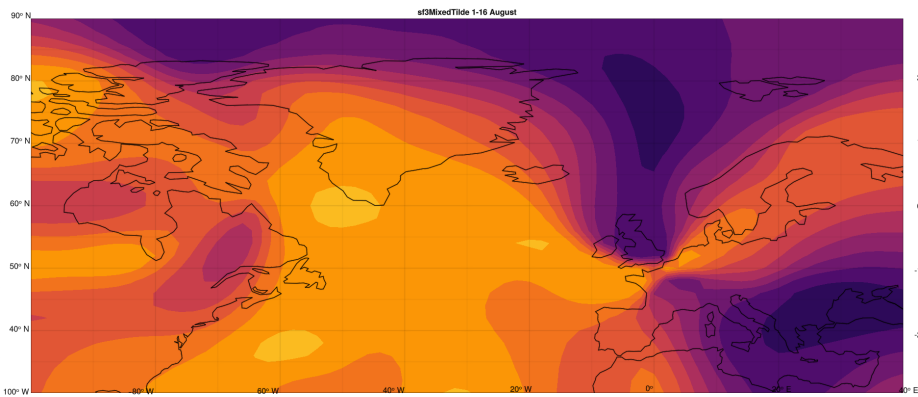
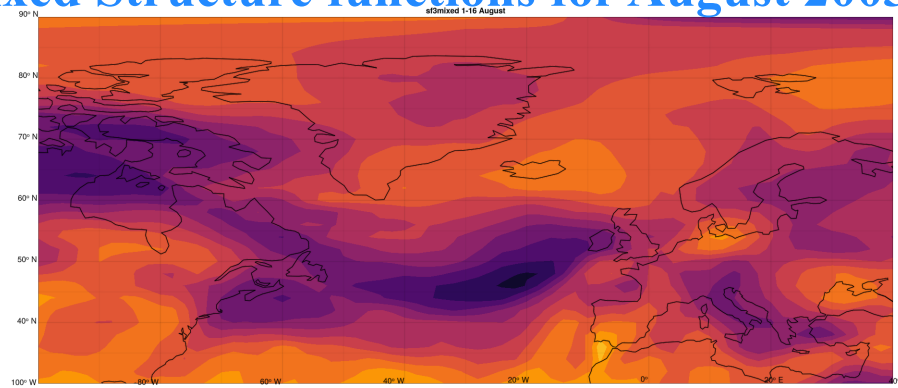


3. Results

Third-order mixed Structure functions for August 2003

3. Results

Third-order mixed Structure functions for August 2003



2. Methodology – IV. Transport equations => Reserve Slides

$$\begin{aligned} \frac{D\overline{\delta\theta^2}}{Dt} + \frac{\partial}{\partial X_\alpha} \left[\overline{\sum \tilde{u}_\alpha \delta\theta^2} + 2\overline{\langle \sum u'_\alpha \delta\theta' \rangle \delta\theta} \right] + \overline{2\delta\tilde{u}_\alpha \delta\theta} \frac{\partial T}{\partial x_\alpha} \\ - \overline{\langle \sum u'_\alpha \delta\theta' \rangle} \frac{\partial}{\partial X_\alpha} \delta\theta + \frac{\partial}{\partial r_\alpha} \overline{\delta\tilde{u}_\alpha \delta\theta^2} + 2\overline{\delta\theta} \frac{\partial}{\partial r_\alpha} \overline{\langle \delta u'_\alpha \delta\theta' \rangle} \\ - \kappa \left[\left(2\frac{\partial^2}{\partial r_\alpha^2} + \frac{1}{2} \frac{\partial^2}{\partial X_\alpha^2} \right) \overline{\delta\theta^2} \right] = -2 \Sigma \end{aligned}$$

$$\begin{aligned} \frac{D\overline{\delta\theta'^2}}{Dt} + \frac{\partial}{\partial X_\alpha} \left[\overline{\sum u'_\alpha \delta\theta'^2} + \overline{\sum \tilde{u}_\alpha \langle \delta\theta'^2 \rangle} \right] + \overline{\langle \sum u'_\alpha \delta\theta' \rangle} \frac{\partial}{\partial X_\alpha} \delta\theta \\ - \overline{2\delta u'_\alpha \delta\theta'} \frac{\partial T}{\partial x_\alpha} + \frac{\partial}{\partial r_\alpha} \left(\overline{\langle \delta u'_\alpha \delta\theta'^2 \rangle} + \overline{\delta\tilde{u}_\alpha \langle \delta\theta'^2 \rangle} \right) \\ - \kappa \left[\left(2\frac{\partial^2}{\partial r_\alpha^2} + \frac{1}{2} \frac{\partial^2}{\partial X_\alpha^2} \right) \overline{\delta\theta'^2} \right] = -2 \Sigma \end{aligned}$$

They indicate the additional forcing exerted by the CM on the random motion

Other terms are to be considered, accounting for the under-resolved scales! (ongoing work)

22 Operators allow for either 2D, or 3D turbulence to be tackled (ongoing work)

Considering the triple decomposition², $\theta = \bar{\theta} + \tilde{\theta} + \theta'$, $u_j = \bar{u}_j + \tilde{u}_j + u'_j$. Where $\bar{\dots}$, $\tilde{\dots}$ and \dots' are respectively the mean, the coherent and the random component.

Scale-by-scale scalar variance budget of CS and random field.

Energy budget of the Coherent motion

$$A_c + D_c + D_{cr} + P_{cm} - P_{cr} - V_c + 2\delta\tilde{u} \overline{\frac{\partial}{\partial r_j} \langle \delta\theta \delta\theta \rangle} + \overline{\frac{\partial}{\partial r_j} \delta\tilde{u} (\delta\theta)^2} = 2\overline{\tilde{\chi}^+} + 2\overline{\tilde{\chi}}.$$

Energy budget of the random motion

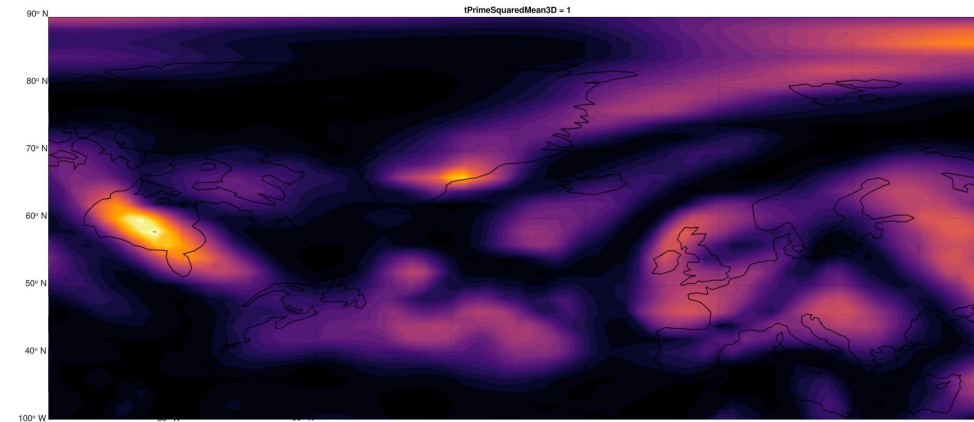
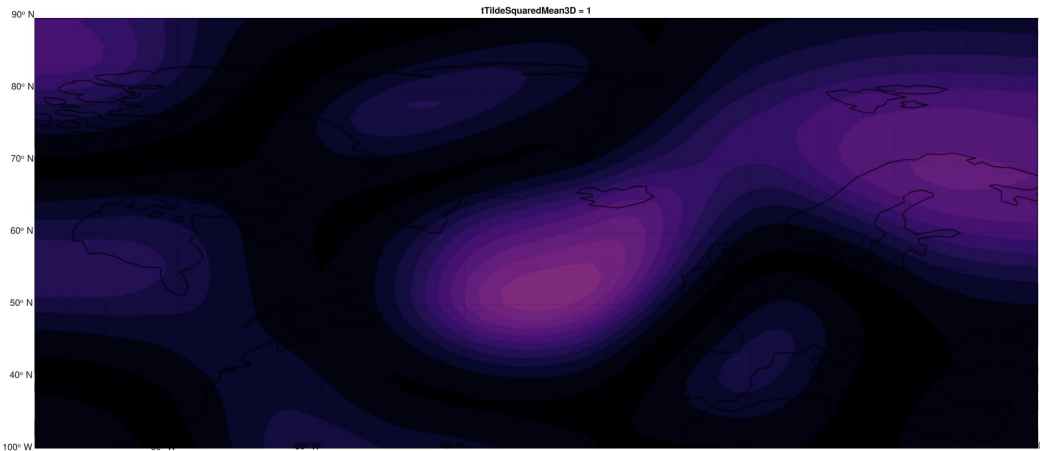
$$A_r + D_r + D_{rc} + P_{rm} + P_{rc} - V_r - 2\delta\tilde{u} \overline{\frac{\partial}{\partial r_j} \langle \delta\theta \delta\theta \rangle} - \overline{\frac{\partial}{\partial r_j} \delta\tilde{u} (\delta\theta)^2} = 2\overline{\chi^+} + 2\overline{\chi}.$$

F. Thiesset, L. Danaila and R. A. Antonia, J. F. M. 2013, 2014

Alves Portela & C. Vassilicos, J.F.M. 2020

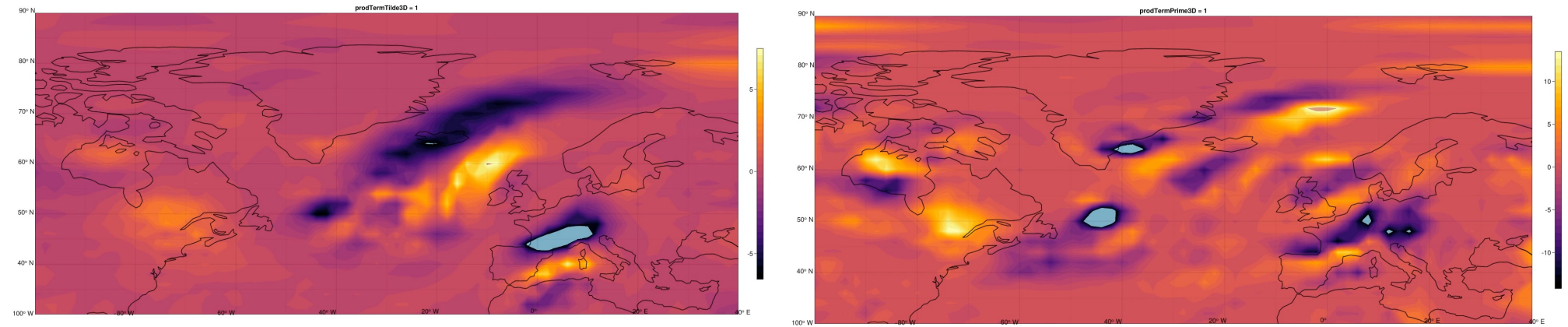
A. Cimarelli et al., J.F.M. 2023

3. Results – II: Variance of Coherent vs. Random



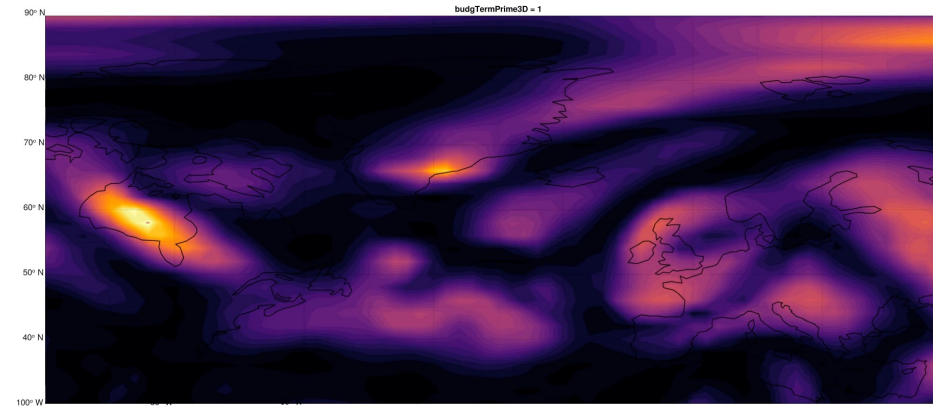
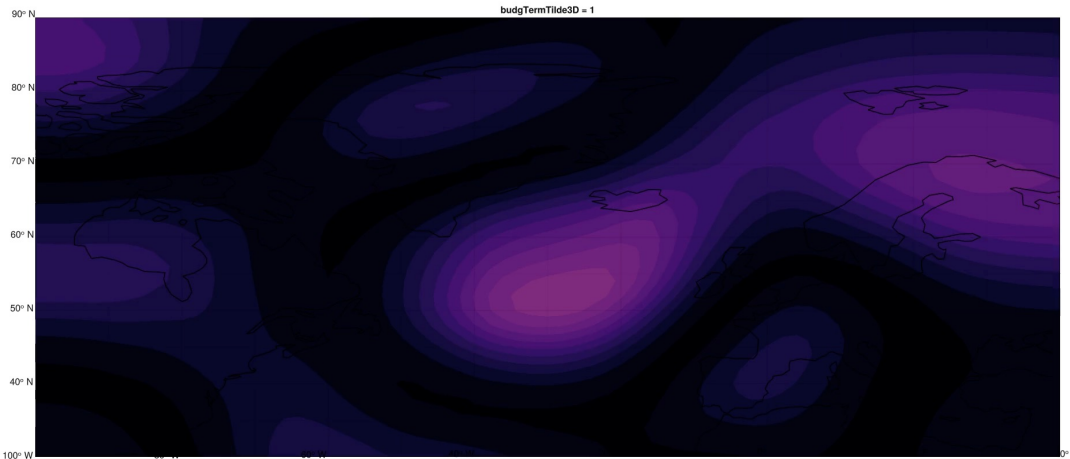
Larger values and more rapid dynamics for RM

3. Results – III: Production of Coherent vs. Random



Larger values and more rapid dynamics for RM
Production which is both positive and negative (sink-like)

3. Results – IV: Temporal derivative Coherent vs. Random



Larger values and more rapid dynamics for RM

3. Results – V: Variance of Coherent vs. Random

Too many slides: pls. consider moving several to Reserve Slides!

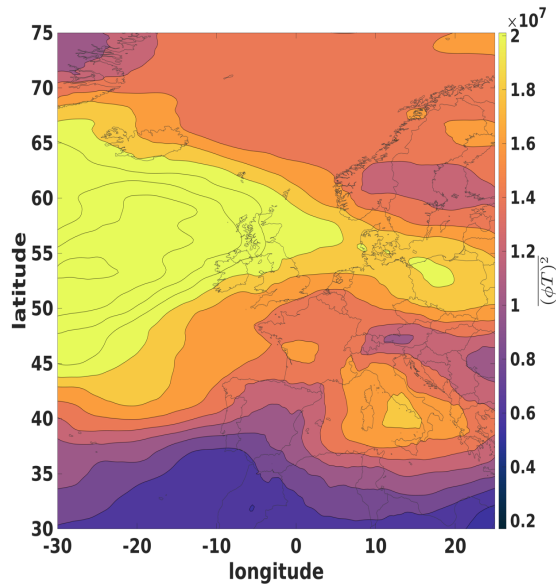
Max number ~ 12–15!!

Use larger type for visibility.

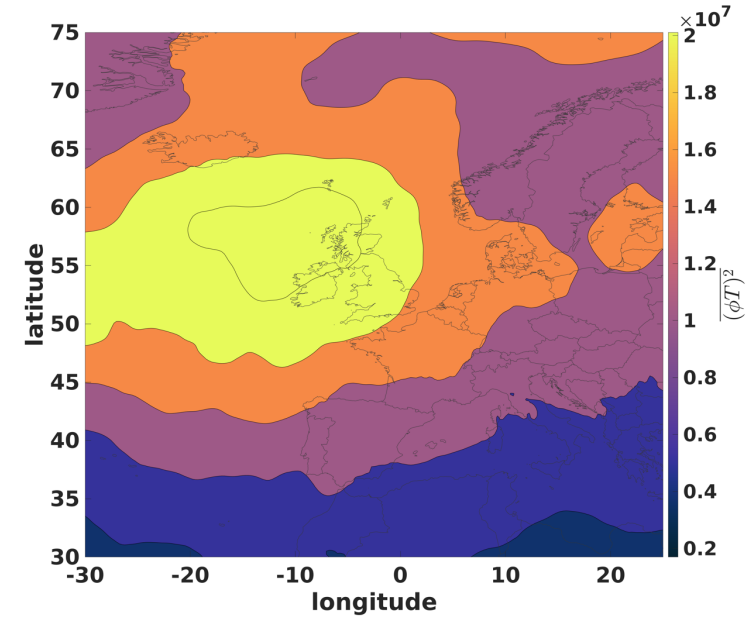
Larger values and more rapid dynamics for RM

2. Results – V. ERA5 data, summer 2003

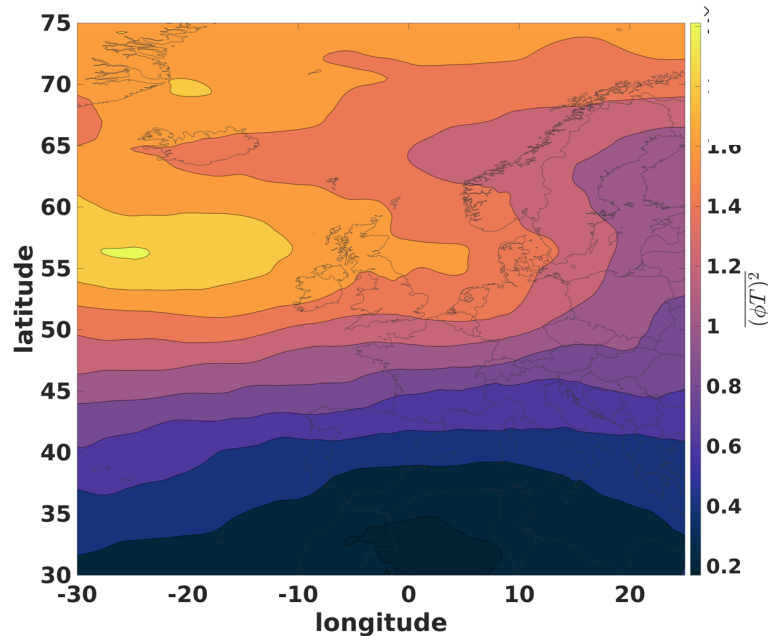
RMS of zonal velocity



RMS of meridional velocity



RMS of temperature fluctuations



Reserve slides
